Write your name here

| Surname | Other names |
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**Pearson Edexcel**
**Level 3 GCE**

Centre Number | Candidate Number

# Further Mathematics

## Advanced Subsidiary
## Further Mathematics options
## 25: Further Mechanics 1
## (Part of options C, E, H and J)

Thursday 17 May 2018 – Afternoon

Paper Reference
**8FM0-25**

**You must have:**
Mathematical Formulae and Statistical Tables, calculator

Total Marks

**Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.**

## Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

## Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- The total mark for this part of the examination is 40. There are 4 questions.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

## Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

*Turn over* ▶







Pearson

Whenever a numerical value of $g$ is required, take $g = 9.8\,\text{m s}^{-2}$ and give your answer to either 2 significant figures or 3 significant figures.

**Answer ALL questions. Write your answers in the spaces provided.**

**1.** A small ball of mass 0.3 kg is released from rest from a point 3.6 m above horizontal ground. The ball falls freely under gravity, hits the ground and rebounds vertically upwards.

In the first impact with the ground, the ball receives an impulse of magnitude 4.2 N s. The ball is modelled as a particle.

(a) Find the speed of the ball immediately after it first hits the ground.

**(5)**

(b) Find the kinetic energy lost by the ball as a result of the impact with the ground.

**(3)**

(a) Let's use suvat to get speed right before it hits the ground.

(+↓) $s = 3.6$ Use formula:

$u = 0$ $v^2 = u^2 + 2as$

$v = v$ Substitute:

$a = 9.8\,\text{ms}^{-2}$ $v^2 = (0)^2 + 2(9.8)(3.6)$

$t$ $v^2 = 70.56 \rightarrow v = 8.4\,\text{ms}^{-1}$ (downwards) M1A1

Impulse is the change in momentum

Formula for change in momentum:

$I = \Delta\text{momentum} = mv_{final} - mv_{initial}$ (mass; velocity)

Substitute into $I = mv - mu$: (↑+)

$4.2 = 0.3(w - (-8.4))$ M1

we take ↑ as positive and before it move downwards ∴ negative

$\frac{4.2}{0.3} = w + 8.4$ A1

$14 - 8.4 = w$

$w = 5.6\,\text{ms}^{-1}$ speed after A1

(b) To get KE lost: $\Delta KE = KE_I - KE_F$

Formula for Kinetic Energy:

$KE = \frac{1}{2}mv^2$ (mass; velocity)

Substitute: $\Delta KE = \frac{1}{2}(0.3)(8.4)^2 - \frac{1}{2}(0.3)(5.6)^2$ M1A1

$= \frac{1}{3}(0.3)[8.4^2 - 5.6^2]$

$= 5.88$ J lost A1

units for energy, Joules

**Question 1 continued**

**(Total for Question 1 is 8 marks)**

2.

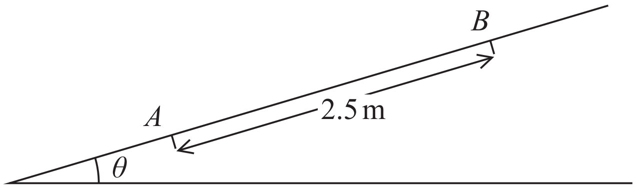


**Figure 1**

Figure 1 shows a ramp inclined at an angle $\theta$ to the horizontal, where $\sin\theta = \frac{2}{7}$

A parcel of mass 4 kg is projected, with speed $5\,\text{m s}^{-1}$, from a point $A$ on the ramp. The parcel moves up a line of greatest slope of the ramp and first comes to instantaneous rest at the point $B$, where $AB = 2.5\,\text{m}$. The parcel is modelled as a particle.

The total resistance to the motion of the parcel from non-gravitational forces is modelled as a constant force of magnitude $R$ newtons.

(a) Use the work-energy principle to show that $R = 8.8$

**(4)**

After coming to instantaneous rest at $B$, the parcel slides back down the ramp. The total resistance to the motion of the particle is modelled as a constant force of magnitude 8.8 N.

(b) Find the speed of the parcel at the instant it returns to $A$.

**(3)**

(c) Suggest two improvements that could be made to the model.

**(2)**





P 6 0 2 0 6 A 0 4 1 6

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Question 2 continued

(a)

★ Work-Energy Principle: an increase of KE/GPE/EPE is caused by an equal amount of positive work done on the body (e.g. engine) and a decrease of KE/GPE/EPE is caused by an equal amount of negative work done on the body (e.g. friction).

★ Remember the work-energy formulae:

Either: WD by force + $KE_i$ + $GPE_i$ = $KE_F$ + $GPE_F$ + WD against friction

(work done, initial kinetic, initial grav. potential, final kinetic, final grav. potential, work lost to friction)

OR: WD by force + $KE_i$ + $GPE_i$ − WD by friction = $KE_F$ + $GPE_F$

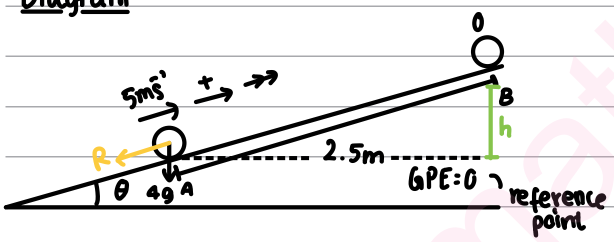
(work done, initial kinetic, initial grav. potential, we subtract this since it leaves the system as heat!, final kinetic, final grav. potential)

★ Formulae for KE and GPE:

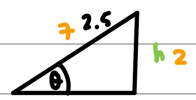
$KE = \frac{1}{2}mv^2$ (mass, velocity)

$GPE = mgh$ (mass, $g = 9.8\,ms^{-2}$, change in height)

Diagram




Get h:




$\sin = \frac{O}{H}$: $\frac{2}{7} = \frac{h}{2.5}$

$h = \frac{5}{7}$

Substitute:

$\frac{1}{2}(4)(5)^2 + 4g(0) - 2.5R = \frac{1}{2}(4)(0)^2 + 4g(h)$ M1A1

Solve for R:

$2(25) - 2.5R = 4g \times \frac{5}{7}$

$50 - \frac{5}{7} \times 4g = 2.5R$ A1

$50 - 8g(\frac{5}{7}) = 2.5R$

$22 = 2.5R$

$R = 8.8$ hence shown A1

(b) We will use the work-energy principle again. As it moves from A to A, ΔGPE = 0 (as it returns to the same height). We only need to consider KE and the work done x2 (R up + down!)

Substitute: $\frac{1}{2}(4)v^2 = \frac{1}{2}(4)(5)^2 - 2(2.5)(8.8)$ M1A1

$2v^2 = 6$

A1 $V = 1.7\,ms^{-1}$ speed when it returns to A





P 6 0 2 0 6 A 0 5 1 6

Question 2 continued

(c) Make resistance variable (e.g. proportional to speed) B1

Don't model the parcel as a particle and consider its dimensions B1

**Question 2 continued**

**(Total for Question 2 is 9 marks)**

3. A van of mass 750 kg is moving along a straight horizontal road. At the instant when the van is moving at $v$ m s$^{-1}$, the resistance to the motion of the van is modelled as a force of magnitude $\lambda v$ N, where $\lambda$ is a constant.

The engine of the van is working at a constant rate of 18 kW. At the instant when $v = 15$, the acceleration of the van is 0.6 m s$^{-2}$

(a) Show that $\lambda = 50$

**(4)**

The van now moves up a straight road inclined at an angle to the horizontal, where

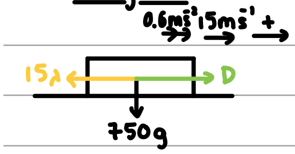
$$\sin\alpha = \frac{1}{15}$$

At the instant when the van is moving at $v$ m s$^{-1}$, the resistance to the motion of the van from non-gravitational forces is modelled as a force of magnitude $50v$ N. When the engine of the van is working at a constant rate of 12 kW, the van is moving at a constant speed $V$ m s$^{-1}$

(b) Find the value of $V$.

**(5)**

(a) Diagram




To get D we will use Power.

Formula for Power:

Power (W) ← $P = Dv$ B1

Driving force (N), velocity (ms⁻¹)

$P = 18$ kW — ×1000 → 18000W

$v = 15$ ms⁻¹ $\qquad 18\,000 = 15D$

$D = D \qquad D = 1200$ N

It's accelerating ∴ use $\Sigma F_x = ma$:

$D - 15\lambda = 750(0.6)$ } Solve for $\lambda$ M1

$1200 - 15\lambda = 450$ A1

$15\lambda = 750$

$\lambda = 50$ hence shown A1





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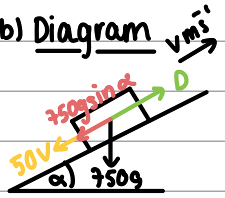
**Question 3 continued**

(b) Diagram




To get D we will use Power.

Formula for Power:

Power (W) ← $P = Dv$ (Driving force (N), velocity (ms⁻¹))

$P = 12\text{kW}$ — ×1000 → 12000

$v = V$

$D = D$

$D = \frac{12\,000}{V}$

As the speed is constant use $\Sigma F_x = 0$ : M1

$D = 750g\sin\alpha + 50V$

$\frac{12\,000}{V} = 50V + 490$ A1

$12\,000 = 50V^2 + 490V$

$0 = 5V^2 + 49V - 1200$ Quadratic A1

Use Quadratic Formula: $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\frac{-49 \pm \sqrt{(49)^2 - 4(5)(-1200)}}{2(5)} = V$

$V = 11.3\text{ms}^{-1}$ speed of van M1A1

Question 3 continued

**Question 3 continued**

**(Total for Question 3 is 9 marks)**

**4.** A particle $P$ of mass $3m$ is moving in a straight line on a smooth horizontal floor. A particle $Q$ of mass $5m$ is moving in the opposite direction to $P$ along the same straight line.

The particles collide directly.

Immediately before the collision, the speed of $P$ is $2u$ and the speed of $Q$ is $u$. The coefficient of restitution between $P$ and $Q$ is $e$.

(a) Show that the speed of $Q$ immediately after the collision is $\frac{u}{8}(9e + 1)$

**(6)**

(b) Find the range of values of $e$ for which the direction of motion of $P$ is not changed as a result of the collision.

**(2)**

When $P$ and $Q$ collide they are at a distance $d$ from a smooth fixed vertical wall, which is perpendicular to their direction of motion. After the collision with $P$, particle $Q$ collides directly with the wall and rebounds so that there is a second collision between $P$ and $Q$. This second collision takes place at a distance $x$ from the wall.

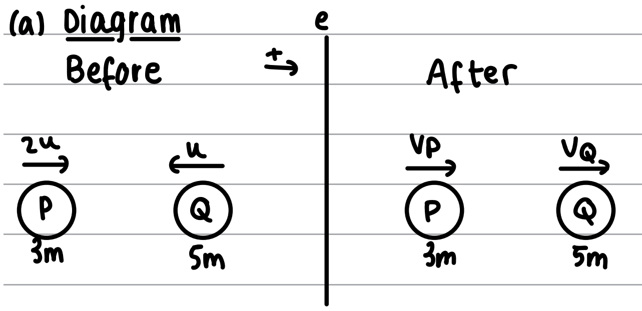
Given that $e = \frac{1}{18}$ and the coefficient of restitution between $Q$ and the wall is $\frac{1}{3}$

(c) find $x$ in terms of $d$.

**(6)**

Question 4 continued

(a) Diagram




We can use the conservation of linear momentum to get this.
conservation of linear momentum means: the total momentum before the collision is the same as the total momentum after. M1

Formula: $m_A u_A + m_B u_B = m_A v_A + m_B v_B$ (initial velocity; final velocity)

Substitute: $3m(2u) + 5m(-u) = 3m v_P + 5m V_Q$

$u = 3V_P + 5V_Q$ Eq1 A1

We can use Newton's Law of Restitution to get an equation.
Newton's Law of Restitution states that: when two objects collide, their speeds after the collision depend on ① speeds before the collision and ② the material from which they're made.

Formula: $e(u_A - u_B) = V_B - V_A$ M1
(coefficient of restitution; initial speed; final speed)

Substitute: $e(2u - (-u)) = V_Q - V_P$

$3eu = V_Q - V_P$ Eq2 A1

Solve simultaneously Eq1 and Eq2: (we're looking for $V_Q$)

$u = 3V_P + 5V_Q$

$3eu = V_Q - V_P$ (×3) $9eu = 3V_Q - 3V_P$ + elimination method

$9eu + u = 8V_Q$

$u(9e+1) = 8V_Q$

$V_Q = \frac{u}{8}(9e+1)$ hence shown M1A1

(b) Let's get $V_P$:

$V_P = V_Q - 3eu$

$V_P = \frac{u}{8}(9e+1) - 3eu$

$V_P = \frac{9eu}{8} - 3eu + \frac{u}{8} \rightarrow V_P = \frac{u}{8}(1-15e)$

For it to not change direction, $V_P > 0$:

$1 - 15e > 0 \rightarrow e < \frac{1}{15}$

As e is a coefficient of restitution, it must be bigger than 0.

$\therefore 0 \leq e < \frac{1}{15}$ Range of e M1A1

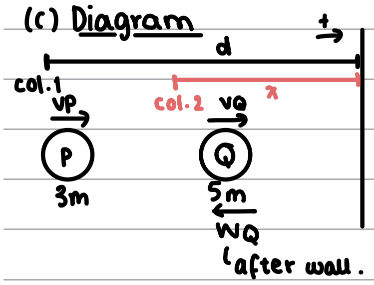


Turn over

P 6 0 2 0 6 A 0 1 3 1 6

**Question 4 continued**

(c) Diagram




P does not change direction as $0 \le e < \frac{1}{15}$

$V_P = \frac{u}{48}$

$V_Q = \frac{3u}{16}$

Get $W_Q$: $\frac{1}{3} \times \frac{3u}{16} = \frac{u}{16}$ (and it changes direction)

$W_Q = \frac{u}{16}$ B1

We will use time to get distance as time $= \frac{\text{distance}}{\text{speed}}$.

For Q, to wall and back:

$$\frac{d}{\frac{3u}{16}} + \frac{x}{\frac{u}{16}} = \frac{16d}{3u} + \frac{16x}{u} = t$$

(to wall; back to x.) time for Q from first to 2nd collision A1

For P, up to point x:

distance $= d - x$

speed $= \frac{u}{48}$

Substitute: $t = \frac{d-x}{\frac{u}{48}} = \frac{48(d-x)}{u} = t$ time for P to get to 2nd collision B1

Equate the times for P and Q (as those are the times taken to reach the same point (the spot of collision 2) they must be equal).

$$\frac{16d}{3u} + \frac{16x}{u} = \frac{48(d-x)}{u}$$ M1

$$16d + 48x = 144(d-x)$$

$$16d + 48x = 144d - 144x$$

$$192x = 128d$$

$$x = \frac{128}{192}d$$

$$x = \frac{2}{3}d$$ x in terms of d

M1A1

**Question 4 continued**





Turn over

**Question 4 continued**

**(Total for Question 4 is 14 marks)**

**TOTAL FOR FURTHER MECHANICS 1 IS 40 MARKS**





P 6 0 2 0 6 A 0 1 6 1 6